Information, finite-dimensional Fock space, and small-scale structure of spacetime

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April, 1994

Abstract

If there exists an upper limit on the amount of information (m, equivalently, entropy) that can be confined in a bounded volume V, then the dimension of the Fock space for any quantum field confined in V is bounded by a corresponding upper limit. Put another way, the upper limit on the dimension of the Fock space exists because states of the field are constrained to have energies bounded by the (critical) Schwarzschild mass of the volume V. We argue that this conclusion about the finite-dimensionality of Fock space is a fundamental consequence of combining quantum field theory with General Relativity (at least within the semi-classical paradigm). Since Fock space is just a representation space for the algebra of the quantum field, and since free-field algebras defined by the canonical commutation and anti-commutation relations do not admit finite-dimensional representations, these algebras need to be modified

to be consistent with a finite dimensional Fock space. Such a modification has consequences both for quantum field theory (in connection with the regularization of the stress-energy tensor) and small-scale structure (if spacetime (since, as we argued elsewhere, spacetime structure is determined by the algebraic and lattice structure of the field algebra). We discuss some of these consequences.